

Logic 04 Reading

Section 1: Contradictory and Contrary Statements

Introduction.

In the last chapter, we dealt with the four kinds of categorical statements: A, E, I, and O statements. In this chapter, we will deal with the first kind of relationship that these statements can have to one another.

There are two relationships categorical statements can have to one another. The first is the relationship of **opposition**. The second is the relationship of **equivalence**. There are four different kinds of opposing relationships and three different kinds of equivalent relationships. In this chapter, we will discuss the first two of the four different relationships of **opposition**.

When we use the term **opposition**, we mean the relationship which we observe in things we call **opposite**. If we say something is the opposite of another thing, we are saying the two things have a relationship of opposition. Statements that are in opposition **affirm** and **deny** the same predicate of the same subject.

There are four ways that any two that any two of these four statements – A, E, I, and O – can be related in opposition. In other words, any one of these statements can be said to be opposite to another in any one of four different ways. They can be **contradictory** to one another; they can be **contrary** to one another; they can be **subcontrary**; and **subalternate**. We will discuss only contradictory and contrary statements in this section and leave subcontrary and subalternate statements until the next section.

_____ **The Rule of Contradiction.** Let us begin the discussion of contradiction by articulating the **Rule of Contradiction**:

The Rule of Contradiction: Contradictory statements are statements that differ in both quality and quantity.

What does this mean?

Remember that we set up a chart to illustrate the four statements in terms of their quality and quantity. You may want to review that chart now.

As you can see, we have shown that each of the four statements show us both its quality and quantity. If we were to take our Rule of Contradiction and apply it, we would look at the chart and ask, “Which are the statements that differ in quality and quantity?”

Consider first the A statement, "All S is P." What is the quality of the A statement? Is it affirmative or negative? Looking at the chart, you see that the quality of A is affirmative. It affirms something about S; namely that it is P.

What is the quantity of the A statement? Is it universal or particular? Does it affirm something about all As or just some of them? In the chart you can see that the quantity of A is universal. It says something about all S's; namely that they are P's.

Therefore, the quality of the A statement is affirmative. The quantity is universal. Which of the other three propositions, the E, I, or O statements are contradictory to A? We know that whichever statement it is has to differ from A in quality and quantity. In other words, it has to be negative (since A is affirmative) and particular (since A is universal). Which of the other three propositions is negative and particular? Look at the chart again, and see that it is the O proposition, "Some S is not P." The O proposition is negative and particular. Since it differs from A in both quality and quantity, it is said to be contradictory to A.

Now let's look at the E statement, "No S is P." What is the quality of the E statements? Negative: it does not affirm, it denies. What is the quantity of the E statement? It is universal, since it denies something of all S's, not just some of them. Therefore, the E statement is negative and universal.

Which of the other statements, A, I, or O, is contradictory to E? In other words, which one of these differs from the E statement in both quality and quantity? Which statement is both affirmative (since E is negative) and particular (since E is universal)? If we look at the chart, we see that it is the I statement. The I statement is both affirmative and particular, making it differ from the E statement in both quality and quantity. So, we can see that the E and I statements are contradictory to one another.

We have determined, then, that A and O statements are contradictory and E and I statements are contradictory.

_____ **The First Law of Opposition.**

The First Law of Opposition: Contradictories cannot at the same time be true nor at the same time false.

In order to further explain this rule, let us look at one pair of contradictories. Let's use the A and O statements.

A: All S is P.

O: Some S is not P.

Is there any way both can be true or both false at the same time? If it is true to say that all S is P, can it also be true to say that some S is not P? No. If all S is P, like statement A indicates, then there can be no S that is not P. But the O statement says that there is some S that is not P. So, they can't both be true.

Furthermore, if it is false to say that all S is P, can it also be false to say that some S is not P? No. If not all S is P, then some S must not be P, just like the O statement says. In other words, if the A statement is false, then the O statement must be true. They cannot both be false at the same time.

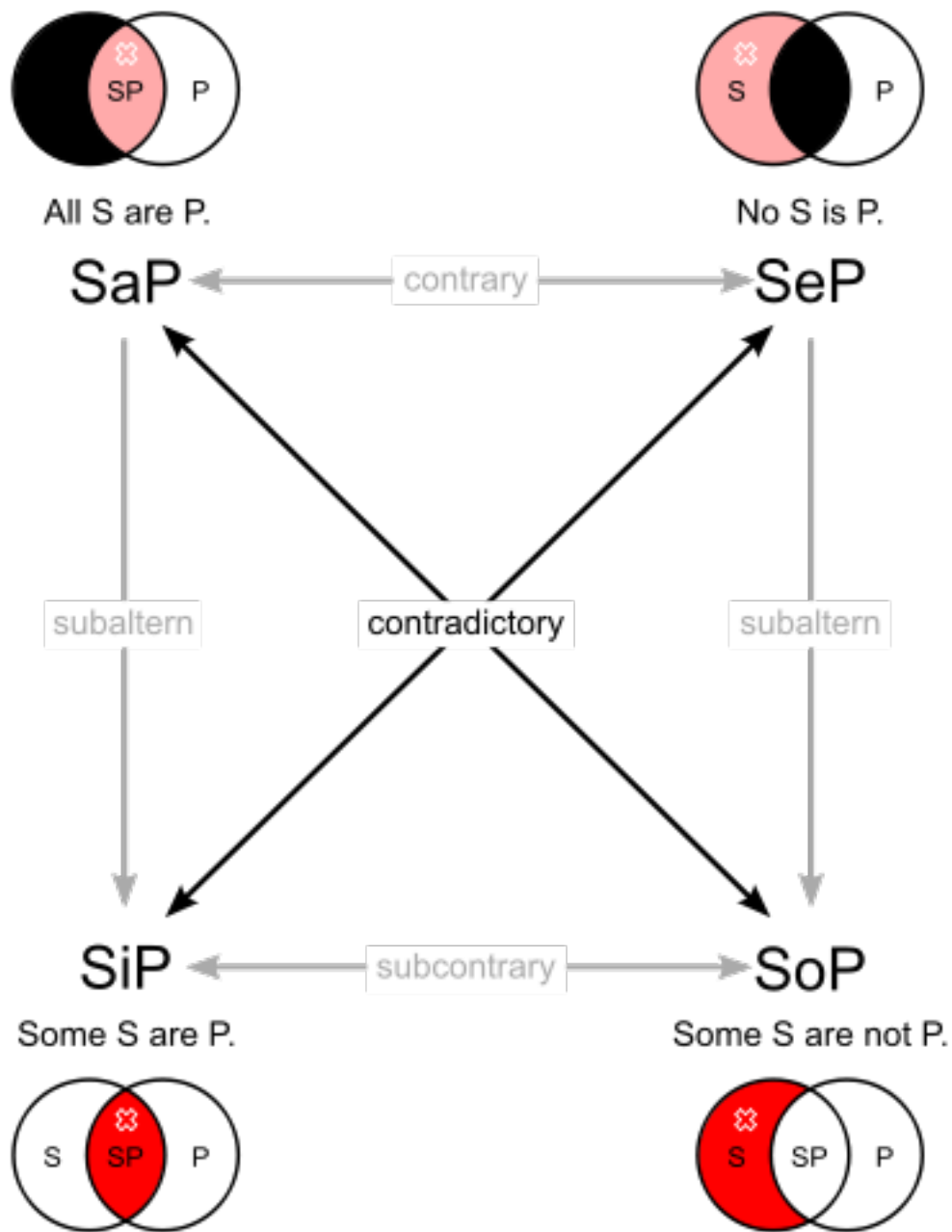
Let's try it on some other statements:

A: All men are mortal.

O: Some men are not mortal.

E: No men are gods.

I: Some men are gods.



The A and E statements, at first seem like they are contradictory. To say “All S is P” seems as if it should be contradictory to “No S is P,” but it is not. A and E statements are opposed to one another, but not by contradiction.

_____ **The Rule of Contraries.** The Rule of Contraries says the following:

The Rule of Contraries: Two statements are contrary to one another if they are both universals, but differ in quality.

Whereas there were two combinations of statements that are contradictory, there is only one combination of statements that are contrary.

We see immediately that the only two statements that are universal are A: “All S is P,” and E: “No S is P.” And although they are universal, one is affirmative and one is negative. They are the same in quantity, but they differ in quality. They must therefore be contrary. You can see the relationship two statements in the chart above.

_____ **The Second Law of Opposition.**

Contraries cannot at the same time both be true, but can at the same time both be false.

If “All S is P” is true, then “NO S is P” must be false. And if “No S is P” is true, then “All S is P” must be false. But if “All S is P” is false, we don’t know whether “No S is P” is true or false, because both could be false at the same time.

If someone told us, for example, “All ducks are white,” and someone else told us, “No ducks are white,” we know it would be impossible for both to be true. But they might both be false. They would both be false if some ducks were white and some were brown. In fact, we know that in the real world, this is in fact the case. Some ducks are white and some are brown.

Section 2: Subcontraries and Subalterns

Introduction.

Remember that in the last section we said that there are four ways that any two of the four statements – A, E, I, and O – can be related in opposition. In other words, any one of these statements can be said to be opposite to another of them in any one of four different ways. They can be ***contradictory*** to one another, they can be ***contrary*** to one another, they can be ***subcontrary***, and ***subalternate***.

_____ **The Rule of Subcontraries.**

The Rule of Subcontraries: Two statements are subcontrary if they are both particular statements that differ in quality.

The difference between contraries and subcontraries is that, while two contrary statements are both universal, two statements that are subcontrary are both particular. Two statements that are subcontrary are both particular, but one is affirmative and one is negative

Of the four statements we have been discussing __A, E, I, and O, which one is particular and affirmative? “Some S is P,” the I statement is particular and affirmative. Of the four statements, which one is particular and negative? “Some S is not,” the O statement, is particular and negative. Therefore, the I and the O statements are subcontrary.

Let’s take a look again at our chart showing the **Square of Opposition**. Which statements are particular but differ in quality? The two bottom statements, I and O.

Like contraries, but unlike contradictories, there is only one combination of statements that are subcontrary. The two contrary statements are A and E. The two subcontrary statements are I and O.

_____ **The Third Law of Opposition.**

The Third Law of Opposition: Subcontraries may at the same time both be true, but cannot at the same time both be false.

If one is false, the other must be true, If one is true, then the other may either be true or false. Look at the following statements:

I: Some S is P.

O: Some S is not P.

If one is false, is there any way the other can be false as well? If one is true, does that require that the other be either true or false? If I say, “Some men are mortal,” and that statement is false, then it would be impossible for the statement, “Some men are not mortal” to be false too. If, “Some men are mortal” is false, then we know that no men are mortal. And if no men are mortal, then, “Some men are not mortal” cannot be false.

_____ **The Rule of Subalterns.**

The Rule of Subalterns: Two statements are subalternate if they have the same quality, but differ in quantity.

They are propositions with the same subject, predicate, and copula, one of which is universal and the other particular.

Unlike contradictories, contraries, and subcontraries, subalterns are really not opposite to one another. But they do have a particular logical relationship with one another that helps to complete the Square of Opposition.

Whereas there was only one combination of statement that we found to be contrary and subcontrary, there are (like contradictories) two combinations of statements that are subalternate.

We see in our chart that there are two pairs of statements that have the same quality but differ in quantity. First, the A statement and the I statement have the same quality (they are both affirmative), but they differ in quantity (the A statement is universal, while I is particular). A and I are subalterns.

Second, the E statement and the O statement have the same quality (they are both negative), but they differ in quantity (one, E, is universal, while the other, O, is particular). E and O are subalterns.

_____ **The Fourth Law of Opposition.**

Subalterns may both be true or both be false. If the particular is false, the universal is false; if the universal is true, then the particular is true; otherwise, their status cannot be determined.

In other words, when it comes to A and I statements, if “Some S is P” is false, then we know that “All S is P” is false. And if “All S is P” is true, then we know that “Some S is P” is true. It also works with E and O statements, since they, too, are subalterns. If “Some S is not P” is false, then “No S is P” is true, then “Some S is not P” must be true.

When we say, in the Fourth Law of Opposition, “otherwise their status is indeterminate,” what we mean is (in the case of A and I statements) if “All S is P” is false, we cannot know whether “Some S is P” is true or false. And if “Some S is P” is true, we cannot know whether “All S is P” is true or false. And (in the case of E and O statements, which are also subalterns) if “No S is P” is false, we cannot know whether “Some S is not P” is true or false, and if “Some S is not P” is true, we cannot know whether “No S is P” is true or false.

If we said, for example, “All ducks are white,” then we know it must be true to say that “Some ducks are white.” But if we know that “Some ducks are white,” we don’t necessarily know that “All ducks are white.” Likewise, if we know that “No ducks are white,” then we also know that “Some ducks are not white” (since some merely means at least one). But if “Some ducks are not white,” we don’t necessarily know that “No ducks are white.”